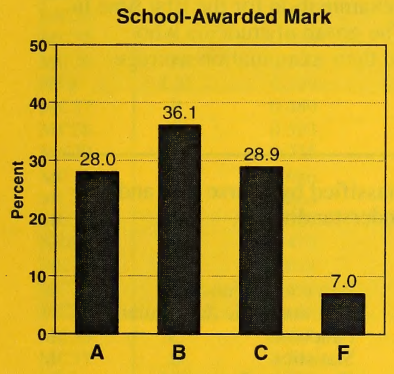
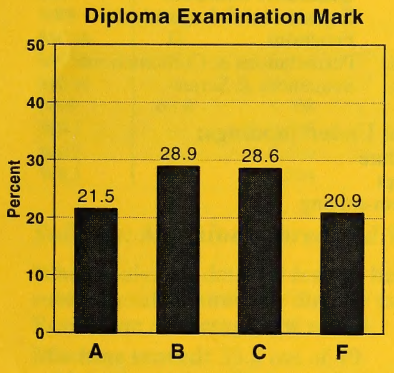
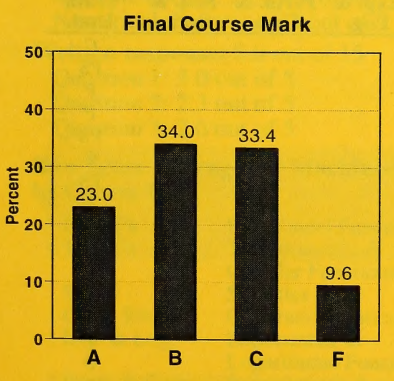
L.1.1131
.2

Mathematics 30

Diploma Examination Results
Examiners' Report for January 1999







The summary information in this report provides teachers, school administrators, and students with an overview of results from the January 1999 administration of the Mathematics 30 Diploma Examination. This information is most helpful when used in conjunction with the detailed school and jurisdiction reports that are provided electronically to schools and school jurisdiction offices. A provincial report containing a detailed analysis of the combined November, January, June, and August results is made available annually.

Description of the Examination

The Mathematics 30 Diploma Examination consists of forty multiple-choice questions worth 57.1%, nine numerical-response questions worth 12.9%, and three written-response questions worth 30% of the total examination mark.

Achievement of Standards

The information reported is based on the final course marks achieved by 9 749 students in Alberta who wrote the January 1999 examination.

- 90.4% of the 9 749 students achieved the *acceptable standard* (a final course mark of 50% or higher).
- 23.0% of these students achieved the *standard of excellence* (a final course mark of 80% or higher).

Overall student achievement in Mathematics 30 was satisfactory. The percentage of students who achieved the *acceptable standard* (90.4%) was almost identical to the percentage for January 1998 (90.6%). Students demonstrated an improved performance on trigonometry and circular functions on the machine-scored section (71.3%) as compared with January 1998 (61.3%). The machine scored results in all other course units was almost the same as for January 1998. Students are generally attempting the written-response questions. The percentage of no responses on the written response were 2.5% for written response 1, 4.0% for written response 2, and 0.7% for written-response 3.

Approximately 49.4% of the students who wrote the January 1999 examination were female. Of these, about 90.0% achieved the *acceptable standard* for a final course mark, compared with 90.8% of the male students. Approximately 20.8% of the female students achieved the *standard of excellence* on the examination, compared with 25.2% of male students.

Alberta
EDUCATION

Provincial Averages

- The average school-awarded mark was 69.6%.
- The average diploma examination mark was 64.4%.
- The average final course mark, representing an equal weighting of the school-awarded mark and the diploma examination mark, was 67.3%.

Approximately 10.7% of the students who wrote the examination in January 1999 and received a school awarded mark had written at least one other Mathematics 30 diploma examination during the January 1998 to January 1999 period. This subpopulation (1 048) achieved an average examination mark of 63.1% compared with 64.5% for the population (8 701) who wrote a Mathematics 30 examination for the first time in January 1999. The group of students who rewrote increased their examination average score by 11.9%.

Results and Examiners' Comments

This examination has a balance of question types and difficulties. It is designed so that students achieving the *acceptable standard* will obtain a minimum mark of 50% or higher, and students achieving the *standard of excellence* will obtain a mark of 80% or higher.

In the following table, diploma examination questions are classified by question type: multiple choice (MC), numerical response (NR), and written response (WR). The column labelled "Key" indicates the correct response for multiple-choice and numerical-response questions. For numerical-response questions, a limited range of answers was accepted as being equivalent to the correct answer. For multiple-choice and numerical-response questions, the "Difficulty" indicates the proportion (out of 1) of students answering the question correctly.

For written-response questions, the "Difficulty" is the mean score (out of 1) achieved by students who wrote the examination.

Questions are classified by course unit and mathematical understanding.

Course Unit:

Poly. Fn.	Polynomial Functions
Trig. Fn.	Trigonometric & Circular Functions
Stat.	Statistics
Quad. Rltns.	Quadratics Relations
Exp. & Log.	Exponential & Logarithmic Functions
Perm. & Com.	Permutations & Combinations
Seq. & Series	Sequences & Series

Mathematical Understandings:

P	Procedure
C	Concept
PS	Problem-solving

Blueprint

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC1	B	0.805	✔							C
MC2	C	0.789	✔							PS
MC3	B	0.561	✔							P
MC4	B	0.853	✔							P
MC5	B	0.350	✔							PS
MC6	C	0.688	✔							C
MC7	A	0.711	✔							PS
NR1	6.0	0.830	✔							P
MC8	D	0.858		✔						P
MC9	D	0.775		✔						C
MC10	D	0.543		✔						P
MC11	B	0.844		✔						P
MC12	A	0.787		✔						C
MC13	B	0.569		✔						P
NR2	6.28	0.819		✔						P
NR3	233	0.495		✔						P
MC14	A	0.759					✔			P
MC15	D	0.947					✔			P
MC16	C	0.716					✔			P
MC17	D	0.893					✔			C
MC18	C	0.844					✔			P

Question	Key	Difficulty	Poly. Fn.	Trig. Fn.	Stat.	Quad. Rltns.	Exp. & Log.	Perm. & Com.	Seq. & Series	Math Und.
MC19	A	0.457					✔			PS
MC20	D	0.634					✔			PS
NR4	2143	0.529					✔			C
MC21	A	0.859				✔				C
MC22	C	0.661				✔				C
MC23	D	0.768				✔				C
MC24	A	0.496				✔				PS
MC25	A	0.550				✔				C
MC26	B	0.700				✔				PS
NR5	4.25	0.509				✔				PS
MC27	B	0.680							✔	P
MC28	C	0.593							✔	PS
MC29	A	0.725							✔	P
MC30	A	0.686							✔	C
MC31	C	0.864							✔	PS
MC32	D	0.607							✔	C
NR6	22.3	0.475							✔	PS
MC33	C	0.874						✔		C
MC34	C	0.790						✔		C
MC35	A	0.666						✔		PS
MC36	C	0.765						✔		C
MC37	B	0.741						✔		PS
NR7	120	0.555						✔		P
NR8	60	0.539						✔		P
MC38	D	0.742			✔					C
MC39	B	0.524			✔					PS
MC40	A	0.703			✔					PS
NR9	63.0	0.708			✔					P
WR1	—	0.608								
WR2	—	0.466								
WR3	—	0.524								

Subtests: Machine Scored and Written Response (Average by Subtest)

When analyzing detailed results, bear in mind that subtest results **cannot** be directly compared. Results are in average raw scores.

Machine scored: 33.8 out of 49
Multiple-choice: 28.4 out of 40
Numerical-response: 5.5 out of 9

Written response: 8.0 out of 15
Question 1: 3.0 out of 5
Question 2: 2.3 out of 5
Question 3: 2.6 out of 5

Raw Score Average for Machine-Scored Questions by Course Unit

Poly. Fn	Polynomial Functions	5.6 out of 8
Trig. Fn	Trigonometric & Circular Functions	5.7 out of 8
Stat	Statistics	2.7 out of 4
Quad. Rltns	Quadratic Relations	4.5 out of 7
Exp. & Log.	Exponential & Logarithmic Functions	5.8 out of 8
Perm. & Com.	Permutations & Combinations	4.9 out of 7
Seq. & Series	Sequences & Series	4.6 out of 7

Raw Score Average for Machine-Scored Questions by Mathematical Understandings*

- Procedural (P): 11.9 out of 17
- Conceptual (C): 12.7 out of 17
- Problem Solving (PS): 9.2 out of 15

* Refer to Appendix E of the 1998–99 *Mathematics 30 Information Bulletin, Diploma Examinations Program* for an explanation of mathematical understandings.

Multiple-Choice and Numerical-Response Questions

The following table gives results for four questions selected from the examination and shows the percentage of students in four groups that answered each question correctly. The comments following the table relate to some of the understandings and skills the students may have used to answer these questions.

Percentage of Students Correctly Answering Selected Machine-Scored Questions

Student Group	Question Number			
	MC 5	MC 10	MC 22	NR 5
All Students	35.0	54.3	66.1	50.9
Students achieving the *standard of excellence* (80% or higher, or A) on the whole examination	55.5	85.5	92.7	85.2
Students achieving the *acceptable standard* but not the *standard of excellence* on the whole examination	31.3	52.6	67.6	48.9
Students who have not achieved the *acceptable standard* (49% or less, or F) on the whole examination	23.7	26.7	34.4	20.9

5. The zeros of two cubic polynomial functions, P and Q, are the same. The numerical coefficients of x^3 are 6 and -3, respectively. If $P(1) = 10$, then $Q(1)$ is

A. equal to -20
***B.** equal to -5
C. equal to 7
D. impossible to determine

The selected questions provide a good indication of student achievement of the curriculum standards in the units Polynomial Functions, Trigonometric and Circular Functions, and Quadratic Relations.

For **multiple-choice question 5** most students achieving the *standard of excellence* answered this question correctly. However, most students achieving the *acceptable standard* but not the *standard of excellence* incorrectly chose D as the answer.

Multiple-choice question 5 was the most difficult question on the exam, only 35.0% of students selecting the correct answer. It may be helpful to examine the steps students were expected to work through to answer this question.

If the zeros and the degrees of both polynomial functions P and Q are the same, then the functions are from the same family and can be defined as:
$P(x) = a(x - l)(x - m)(x - n)$
$Q(x) = k(x - l)(x - m)(x - n)$

Since the numerical coefficient of x^3 in P is 6, $a = 6$, and since the numerical coefficient of x^3 in Q is -3, $k = -3$.

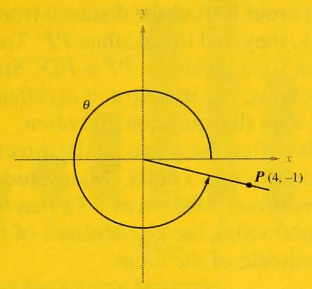
$P(x) = 6(x - l)(x - m)(x - n)$
$Q(x) = -3(x - l)(x - m)(x - n)$

$P(1) = 10$ so we know that
$6(1 - l)(1 - m)(1 - n) = 10$
so $(1 - l)(1 - m)(1 - n) = 10/6$

$Q(1) = -3(1 - l)(1 - m)(1 - n)$ and since
$(1 - l)(1 - m)(1 - n) = 10/6$
$Q(1) = -3(10/6) = -5$

Use the following information to answer the next question

The point $P(4, -1)$ is on the terminal arm of an angle in standard position, as shown below.



10. The measure of angle θ, correct to the nearest tenth of a radian, is

A. 0.2 rad
B. 3.4 rad
C. 5.8 rad
***D.** 6.0 rad

For **multiple-choice question 10**, many students chose solution C or 5.8 radians. This means that the students correctly determined values for both the reference angle 14° and its companion rotational angle of 346°. However, instead of converting the 346° to radians, students took an incorrect step and subtracted 14° from 346° and then converted this result to a radian measure. Students either lost track of the fact that the necessary subtraction from 360° had already been done, or they did not fully understand the concept of reference angle. This question was answered correctly by over 85.5% of the students achieving the *standard of excellence*. However, only about half of the students achieving the *acceptable standard* but not the *standard of excellence* answered it correctly.

Use the following information to answer the next question

The following four equations, when graphed, are four different conics.

I	$y^2 + 12x - 60 = 0$
II	$x^2 + y^2 - 25 = 0$
III	$25x^2 + 9y^2 - 225 = 0$
IV	$16x^2 - 25y^2 - 400 = 0$

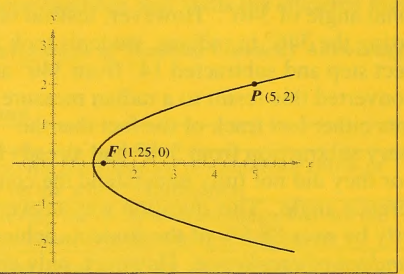
22. The equation above that satisfies the locus definition that states that the sum of the distances from any point P on the curve to two distinct fixed points is always constant is equation

A. I
B. II
***C.** III
D. IV

Multiple-choice question 22 clearly distinguished between the students who achieved the *standard of excellence* on the examination, those who achieved the *acceptable standard* but not *the standard of excellence*, and those who did not achieve the *acceptable standard*. Of the students who achieved the *standard of excellence*, 92.7% answered the question correctly. For this question, 67.6% of the students who achieved the *acceptable standard* but not the *standard of excellence* answered the question correctly; whereas, only 34.4% of the students who did not achieve *the acceptable standard* answered the question correctly. In order to obtain the correct answer, students needed to recognize the locus definition of an ellipse and identify equation III as the equation of an ellipse. Many students who did not achieve the *acceptable standard* incorrectly chose equation II as the solution. This shows that either the students did not recognize the definition as an ellipse, or they could not distinguish between the equation of an ellipse and the equation of a circle.

Use the following information to answer the next question

The graph of $y^2 - x + 1 = 0$ is shown below. The focus of the parabola is $F(1.25, 0)$.



Numerical Response

5. The distance from point $P(5, 2)$ to the directrix, correct to the nearest hundredth, is _____.

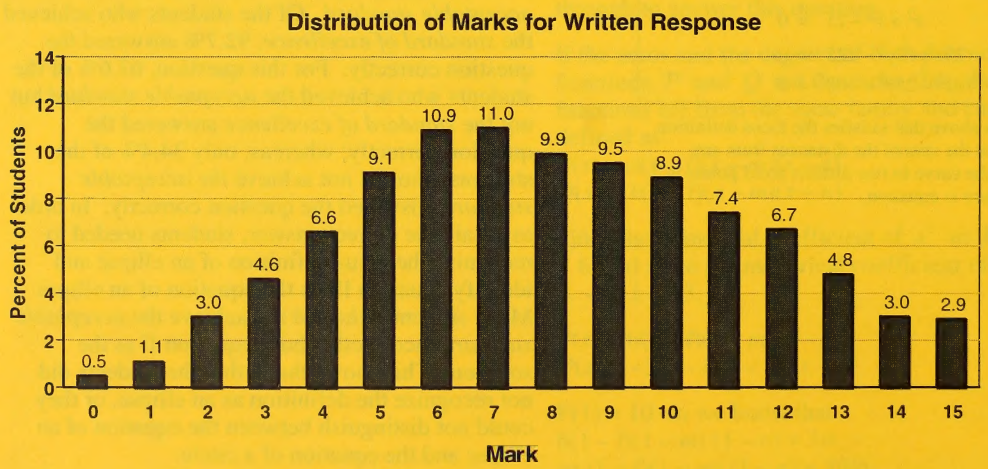
(Record your answer in the numerical-response section on the answer sheet.)

Answer: 4.25

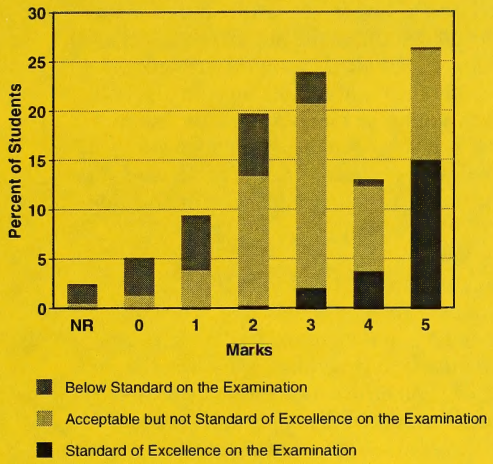
For **numerical-response question 5**, students who achieved the *standard of excellence* performed well on this question; 85.2% answered it correctly. In order to find the distance from P to the directrix, they had to calculate PF. They recognized that for a parabola $PF = PD$. Students who did not achieve the *standard of excellence* had difficulty with this question and often, incorrectly, identified the y-axis as the directrix and gave the distance as 5 units. Many students also gave an incorrect solution of 3.75 that they obtained by subtracting the x-coordinate of point P from x-coordinate of the focus.

Written-Response Questions

The graph below shows the percentage of students achieving various marks on the written-response questions. The maximum mark obtainable was 15. Of the students who wrote the examination, 0.5% received no marks for the written-response questions, 53.1% received 8 marks or *more (acceptable standard)*, and 17.4% received 12 marks or more (*standard of excellence*) out of 15.



Distribution of Marks for Question 1

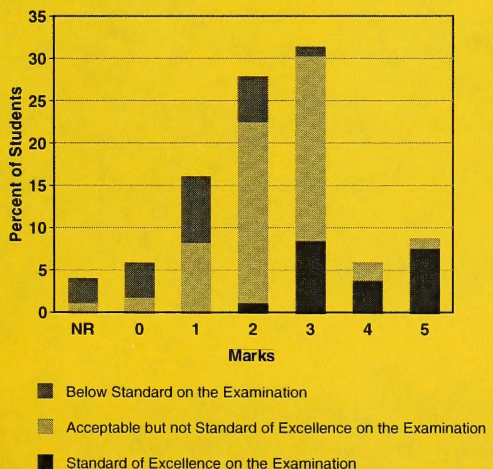


Question 1 required students to determine the annual growth rate of the number of phone numbers in use in a small Alberta community for five consecutive years. Many students incorrectly calculated the growth rate by dividing the number of new phone numbers assigned by the total number of phone numbers in use in the same year as opposed to the previous year.

The students were also required to determine an exponential function that would approximate the number of phone numbers in use for any year after 1989. Overall, this part of the question was fairly well done; the most common error was not adding one to the growth rate in the function.

Students were also asked to find the total number of phone numbers possible in the community. They were also asked to use this number of possible phone numbers and the exponential function they found previously to predict the year in which a new exchange would be required. Students, generally, did an excellent job of calculating the total possible number of telephone numbers but their explanations of their assumptions often lacked clarity.

Distribution of Marks for Question 2



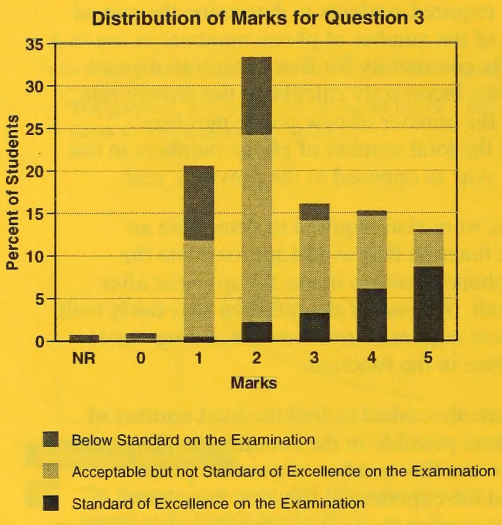
Question 2 required students to analyze information about a sine function and sketch the graph of the function that would satisfy the given conditions. Students were also required to find the equation of the function and algebraically determine the x-intercepts of the graph.

Many students misinterpreted the points of intersection of $y = f(x)$ with the line $y = 1$ as the x-intercepts of the function. This misinterpretation caused them to sketch an incorrect graph.

Students exhibited a good understanding of the parameters a, b, c, and d in the equation $f(x) = a \sin b(x + c) + d$. Some markers commented that students' understanding of 'b' may not have been strong but because $b = 1$, they would get credit even if they did not bother to calculate a value.

Some of the better students drew an incorrect graph but when they algebraically determined the x-intercepts, they were able to say that their graphs did not support their solution.

National Library of Canada
Bibliothèque nationale du Canada
3 3286 51905166 4



Question 3 required students to count the number of cards used to build a 'house.' It also required them to find a pattern and determine the number of cards used for a larger 'house.' Students were then asked to consider the general case and determine an algebraic expression for finding the number of cards required for a house of k levels. Finally, students were asked to find the minimum number of decks of 52 cards that could be used to build a similar house with no cards left over.

Students who did not meet the *acceptable standard* did not find a pattern in the house of cards. They also confused t_n and S_n in the second bullet. These students were unable to generalize; they tried Pascal's triangle, nCr, or geometric series formulas to find a pattern. Students who met the *acceptable standard* but not the *standard of excellence* understood the graphical pattern and were often able to find the correct answers to the second and fourth bullets by repeated calculations. They tried to find an expression in the third bullet but were unsuccessful.

Students achieving the *standard of excellence* used a variety of unique strategies including sigma notation and recursive formulas. They generally used a systematic trial and error method to solve the last bullet.

For further information, contact Kathy McCabe (kmccabe@edc.gov.ab.ca) or Corinne McCabe (cmccabe@edc.gov.ab.ca) at the Student Evaluation Branch at (780) 427-0010. To call toll-free from outside of Edmonton, dial 310-0000.

Copyright 1999, the Crown in Right of Alberta, as represented by the Minister of Education, Student Evaluation Branch, 11160 Jasper Avenue, Edmonton, Alberta T5K 0L2. All rights reserved.

Alberta educators** may reproduce this document for **non-profit educational purposes